What Paradoxical Statements really are

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The most simple example of a paradoxical statement seems to be

S: This statement S is lying.

If we assume **S** is lying, then **S** is not lying; but if we assume **S** is not lying, then **S** is lying. So, what is wrong here?

The problem here lies in what we think a statement is: Under a formal point of view, a statement may be ***true***, may be ***false***, or may be ***empty*** (undefined). The statement

Alice is pregnant

for example cannot be assigned a truth value (***true*** or ***false***) as long as we do not know of which person Alice exactly we are speaking. So, at least now, it is an ***empty*** statement.

We see: Depending on the context **c** in which a statement **X** occurs, the statement's truth value v(**c**,**X**) may be ***true***, ***false***, or ***undefined***. The context in this sense is at least a point in time, but may also contain definitions of the objects the statement refers to.

Formal logic usually ignores this fact, and so mathematicians see **S** as a problem. I do not:

Paradoxical statements are statements **X** for which there is no context **c** such that v(**c**,**X**) is different from ***undefined***.

Now we prove that the statement **S** given above is paradoxical in the sense of this definition. If not, a context **c** would exist, such that v(**c**,**S**) is ***true*** or ***false*** and is solution of the equation

$$v(\mathbf{c},\mathbf{S}) \;=\; v(\, v(\mathbf{c},\mathbf{S}) == \textbf{\textit{false}}\,)$$

In other words: One element **x** of { ***true***, ***false*** } would be solution of the equation

$$\mathbf{x} \;=\; v(\, \mathbf{x} == \textbf{\textit{false}}\,)$$

This equation however is satisfied neither by ***true*** nor by ***false*** (read == as "is identical to").

What we have learned is: The fact that paradoxical statements exist is telling us that to have only two truth values (***true***, ***false***) is not enough; we also need a value ***undefined***.

We also see that mathematical logic should be generalized to

no longer ignore the fact that mapping statements to boolean values is not enough:

Reality is mapping statements *context-sensitive* to { ***true***, ***false***, ***undefined*** }.

The fact that mathematical logic as you find it in text books is ignoring context (especially time) and knows only two boolean values, may be a consequence of the fact that at least mathematical statements can only be ***true*** or ***false*** and are so independent of the point in time we look at them:

Mathematical laws are statements valid in every context.

Other statements however do not necessarily have this nice feature – and this may be so even for statements that describe physical laws: Researchers believing in String Theory are no longer sure that our universe is the only one, and they tell us that, should other universes exist, the physical laws valid there may differ from the physical laws valid in our universe.

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