Fig. 9.4 Rotations between three wave functions

The nine dimensions suggest that there are nine possible turns. We would have turns from 1 to 2, 2 to 3, and 3 to 1. Then we have the same three turns backwards. That leaves us with three self-turns. However, in Fig. 9.4, how can the red arrow make a 360° turn? You can do that in two ways: via the blue dimension or via the green dimension. The self turns are related, just like in the case of SU(2). But now we have two possible self-turns instead of three. That means that we have in fact 8 dimensions instead of nine. Hence, this symmetry is called SU(3) , which is 8 dimensional. It is the Special version of U(3), which is 9 dimensional. It describes how three waves can look alike and can turn into each other.

An example are quarks and gluon s. Quark s are the particles that for instance protons and neutrons are made of. Gluons are the particles that keep quarks together in e.g. a proton. They are responsible for the strong force. We will get to that later. Each quark and each gluon can be in one of three states. Each state is represented by a colour (red, green and blue). Off course they do not actually have a colour, but this is a way to describe the three states they can be in. They can change state at free will. Hence, this is an internal symmetry or an internal degree of freedom. You can compare it with the Hulk that now has three faces to choose from. It can change from rosy red to green (as you know from the movies), but now also into blue.

This example shows that SU(3) is related to the strong force.

9.2 The Electromagnetic Field

The first symmetry we will look at is the U(1) symmetry. The others will come back later. I explained that a U(1) operation leaves the world unchanged and is therefore a symmetry of the universe. But I also promised that nothing is as it seems in our wavy quantum world…

You see, the problem is this: If I just take a wave in the field (massive or not), it is symmetric with respect to a global phase change . If I apply a phase change throughout the universe in one go, I find that nothing measurable will change. *But when the speed of light is the maximum velocity, how can I change the phase in the entire universe in one go?* The vacuum must propagate the phase shift and cannot do that in any way faster than its elasticity permits. But such a propagation of a phase shift is not described in the waves and potentials we have so far. So something is missing.

Let's first consider what we are dealing with here. What happens when a phase shift takes place? It leads to a sudden change in the field strength. This "shock to the system" was supposed to be everywhere at the same time. But since this is impossible, we must now see how to propagate such a phase shift (see Fig. 9.5). You could compare the phase shift to throwing a stone in the water. It leads to a wave propagating outwards. The phase shift will do the same. The phase shift leads to a phase shift a little further in space. That phase shift will lead to a phase shift still further down. And so on at the speed of light.

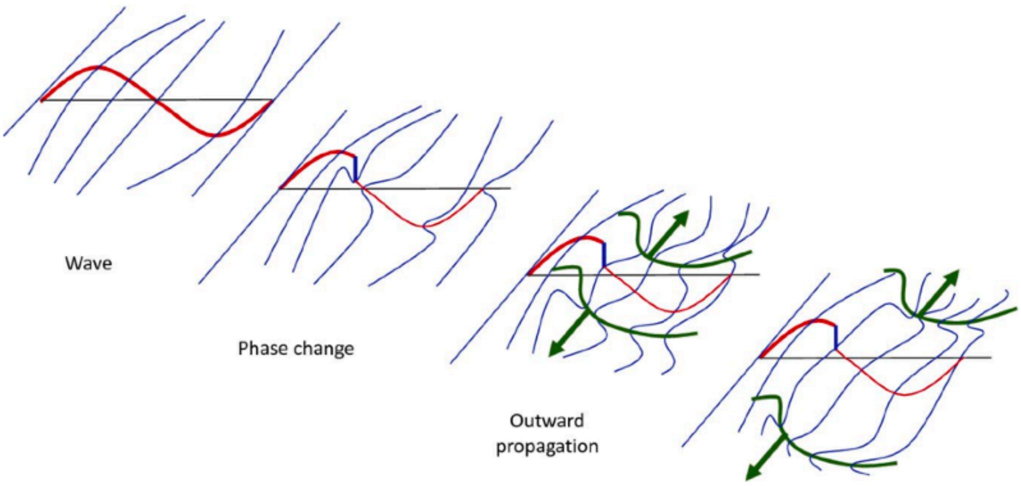

Fig. 9.5 Propagating a phase shift

Why at the speed of light? When you throw a stone in the water, an excitation is created in a field. So the phase shift creates an excitation in a field. At this point of our reasoning it is not clear what field is excited. Let's assume for now that this is a field that does not connect to Higgs and is therefore massless. Then the propagation of the phase shift happens at light speed.

Suppose the phase shift happens in an electron field , i.e. the field that contains an electron wave when it gets excited. The phase shift itself does create a wave, but it cannot be an electron wave. If it were, an electron would be spraying of electrons all the time (with each phase shift a new electron would be created). Clearly there is not enough energy to do such a thing. So it must be a different type of wave resulting from a phase shift. But the electron field itself cannot harbour another type of wave. It can only contain excitations that are electron quanta. *So a phase shift in a field does not create an excitation in that same field.* Consequently, the phase shift must be propagated in a *different field.* A field that gets created special to propagate a phase shift.

Summarizing, the problem is that a global symmetry is not possible due to relativity, or due to the elasticity of the vacuum. A symmetry operation needs to be global. When it cannot be global (in one go for the entire universe), it must be propagated. Hence we say that the symmetry can only be a local symmetry . Therefore, we must add a field that is able to propagate the symmetry operation in line with the elasticity of the vacuum.

The field that is generally added to make a symmetry local is called a Gauge field . The excitations in such a field are called Gauge bosons . They generally turn out to be bosons. We will treat the properties of bosons later.

How can we describe such a Gauge field? When a phase shift happens it creates a difference with the neighbouring part of the field that has not been shifted yet. *This difference is in essence a potential difference.* That potential difference gets propagated away by the gauge field. It is important that the field that produces the phase shift is “connected” to the gauge field. Being connected means that the potential difference of the phase shift is picked up by the gauge field. When it is picked up, it is the same as lifting up a rope in the Gauge field. This sets off a wave in the Gauge field (see Fig. 9.6).

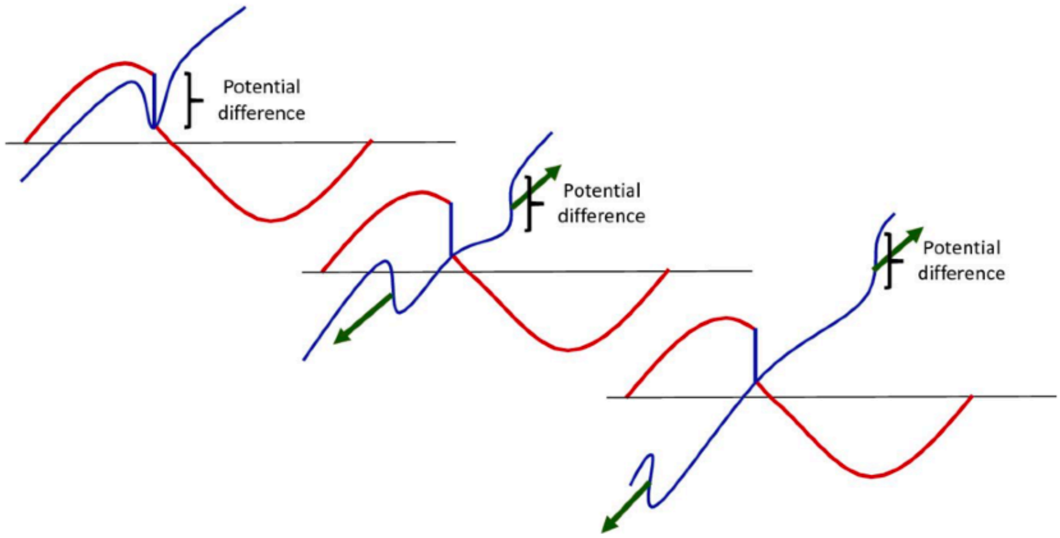

Fig. 9.6 A Gauge wave as a result of a phase shift

Such a wave has all the things we expect from a wave in a field: it has a basic wave expression that describes the dynamics of the wave: an oscillation between potential difference and movement in the field. It can have a mass potential when the Gauge field would be connected to Higgs. We will see later why the U(1) Gauge field is not connected to Higgs (see Chap. 16).

The potential it propagates is called the coupling potential or interaction potential . Basically, this potential (or spring) describes the connectedness of the field that shifted phase and the Gauge field that propagates the shift away. When the potential is felt by another field, it looks like Hooke's law again! (see Fig. 9.7).

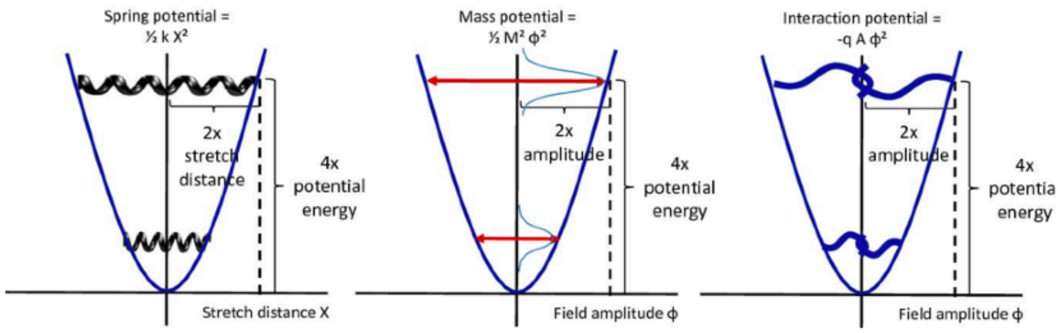

Fig. 9.7 The coupling- or interaction-potential has similar properties as Hooke's law and the mass potential. The interaction potential can be seen as behaving like a spring

In this potential q describes the connectedness. It is also called the coupling constant . A is the potential (or field strength) of the Gauge field and Φ is the field strength of the field that created the phase shift. This looks logical: the energy exchange between the field and the Gauge field depends on their field strengths as well as the connectedness between the fields. Therefore, the spring strength is equal to 2qA in this case. So the strength of the spring that is felt by the field φ is determined by the coupling constant q and the amplitude A of the gauge field. This determines the potential energy that is felt by the field φ. Hence, the stronger the coupling the stronger the potential is felt and the stronger the gauge field, also the stronger the potential is felt. This potential describes how the wave gets set off. It describes how the Gauge field gets excited as a consequence of the phase shift.

This potential also says something else: what happens when a propagating phase shift meets another field quantum? E.g. it meets a proton quantum in a proton field. Then this potential creates a "spring" that is felt by the proton since the proton field is connected to the Gauge field. The potential felt is a combination of both field strengths and their connectedness. So here we see the basics appear of a force! We will get back to how the force mechanism works in Sect. 9.4. We will show how the result of this process is a change in velocity and direction of the quanta involved and how "attracting" or "repelling" works as a consequence of this.

Mathematically, when this is worked out for the U(1) symmetry we get an interesting result. *The basic wave and the coupling potential could describe the electromagnetic field!* Not only does this fit well with the field descriptions of the E-M field, it is also the simplest fit. Electromagnetism is the simplest gauge theory [Ref. 8, p. 129]. So the electromagnetic field could actually be the Gauge field that is needed to make phase shifts symmetry local. Hence, in quantum electrodynamics it is assumed that it is. Consequently, when a charged particle shifts phase, it creates a photon .

Taking this further, you could say that when the speed of light would be infinite, the phase shift symmetry would actually be a global symmetry and the gauge field needed to propagate the phase shift would not exist. I.e. the electromagnetic field would not exist if the speed of light would be infinite. Consequently, you could say that the electromagnetic field is a direct result of the elasticity of the vacuum!

Now we can recognize the variables in the coupling potential: q is the electric charge ! And A is the electromagnetic potential . You may be familiar with the electric field E and the magnetic field B in the electromagnetic theory. However, E and B are linked. An electromagnetic wave is a continuous oscillation between E and B. It turns out that E and B can also be described by one potential, generally denoted by A.

A phase shift can take place in two directions. In Fig. 9.2 the phase changes from 100° to 170°. It changes forward, or in Fig. 9.1 to the right around the circle (see Fig. 9.8). It could also change backward, or in Fig. 9.8 to the left around in the circle. A phase shift going forward (right around) is associated with a positive charge, while a phase shift in the opposite direction is associated with a negative charge.

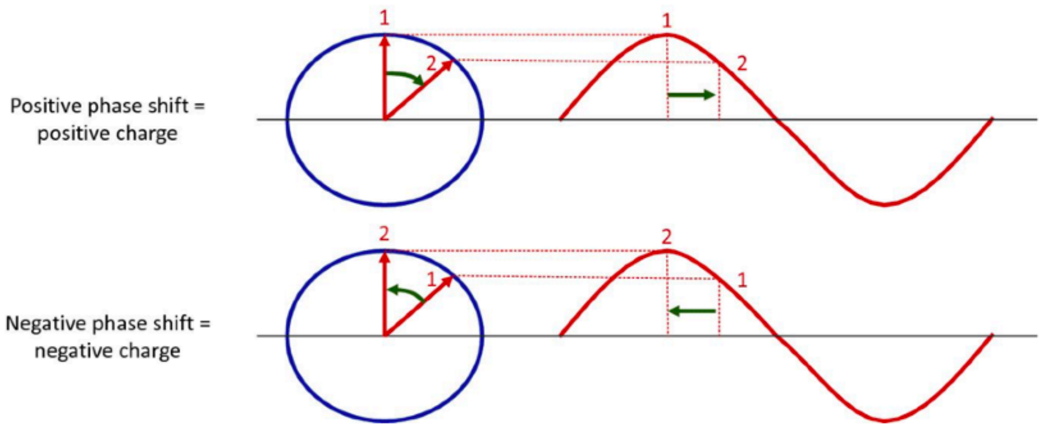

Fig. 9.8 Positive and negative charge are just phase shifts in opposite direction

So with q being the electric charge, it is clear what its role is: the “charge of a particle” determines if and how it feels the electromagnetic field. The charge determines whether and how a wave creates or experiences a phase shift. The way a wave (a quantum, a particle) creates a wave in the gauge field when it shifts phase is exactly the same as the way it experiences a propagating phase shift from elsewhere!

We said before that a global phase shift does not change the energy or momentum. This is the reason it is a symmetry in the first place. Clearly, when this symmetry can only be local this changes: electromagnetic waves do carry energy and momentum.

The gauge wave can store information in the wave about energy (average frequency), momentum (average wavelength) and the potential it propagates away (the phase change). It spreads throughout space-time and so it carries information about position in space-time. It has a spin orientation, and so it carries spin information. There is no room for other information to carry in the wave. So it cannot carry a charge. A charge (the ability to produce and feel phase shifts) would require the wave to hold information about the type of charge (negative or positive). The photon holds only one number besides the regular space-time, spin, momentum and energy information one finds in a wave. This number is the phase change, a potential. The gauge group U(1) is abelian . This means that it does not matter in what order phase changes take place, the result is always the same. Put differently, the photon has no (hidden) memory of the way it was created by the phase shift. It does not know whether it is the first phase change, the second or the third, nor what phase changes have come before.

9.2.1 QED

When we put the waves and potentials we met so far together we get a landmark equation: the equation that describes the waves and potentials for QED (Quantum Electro Dynamics). It tells us how charged particles move in an electromagnetic field. This is a very well tested theory and extremely successful: the measurements agree with the theory to sometimes extremely high precision [e.g. Ref. 60, p. 196; Ref. 30, p. 641]. In the equation you find two types of fields and it describes the waves and potentials for those fields (see Table 9.1).